A Robust Stability Methodology for Track Following Servo Systems

Mohammadreza Chamanbaz, *Member, IEEE*, Fabrizio Dabbene, *Senior Member, IEEE*, Roberto Tempo, *IEEE Fellow*, Venkatakrishnan Venkataramanan, *Senior Member, IEEE* and Qing-Guo Wang, *Member, IEEE*

Abstract— **In this paper, we use the Youla parameterization for an uncertain LTI system to characterize the set of all stabilizing controllers. Next, this set is used as a search space to find the controller which minimizes $\mathcal{H}_2$ norm of the transfer function from disturbance to output. Finally, this methodology is used for stabilization of a 8-th order plant affected by parametric uncertainty describing a voice coil motor (VCM) of a hard disk drive (HDD).**

Index Terms— Disk Drive Servo Design, Parametric Uncertainty, Youla Parameterization.

I. INTRODUCTION

Uncertainty has long been a critical issue in control theory and applications. There are a number of approaches to tackle uncertainty in dynamic plants. Recently, the design based on probabilistic robust theory and randomized algorithms has gained considerable attention in the control community. This approach benefits from randomization in the uncertainty space and (convex) optimization in the design parameter space [1]. Since in a probabilistic framework the performance index (including stability) is guaranteed to hold only in probabilistic sense, the question that may arise is how stability of the closed-loop system can be robustly guaranteed. Indeed, in most applications, the stability of closed-loop systems is a hard constraint which should not be violated even for a set of uncertainty having small probability measure.

The present paper is motivated by this problem. We are using the classical $Q-$Parameterization (Youla parameterization) in order to find the set of all stabilizing controllers for the uncertain plant. Next, the original problem (which is minimizing the $\mathcal{H}_2$ norm of the transfer function from disturbance to output) is reformulated as designing a transfer function Q such that the $\mathcal{H}_2$ norm of the closed-loop system is minimized. The possibility of designing a destabilizing controller is eliminated by posing constraints on Q in terms of stability and $\mathcal{H}_\infty$ norm bound. Therefore, the resulting controller achieves robust stability along with nominal performance. Some technical details regarding the design procedure are given in a technical report available upon request.

M. Chamanbaz and V. Venkatakrishnan are with Data Storage Institute, Singapore (emails: {Mohammad_C, Venka_V}@dsi.a-star.edu.sg).

R. Tempo and F. Dabbene are with IEIIT-CNR Torino, Italy (emails: {roberto.tempo, fabrizio.dabbene}@polito.it).

M. Chamanbaz and Q-G. Wang are with Department of Electrical and Computer Engineering, National University of Singapore (email: elewqg@nus.edu.sg).

II. PROBLEM FORMULATION

The voice coil motor (VCM) of a hard disk drive is considered to be in the form

$$G_{VCM} = \sum_{i=1}^{4} \frac{A_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{1}$$

where ζ_i, ω_i and A_i are damping ratio, natural frequency and modal constant respectively. Throughout this paper, we assume natural frequency and damping ratios to vary within 5% and 10% from their nominal values. Therefore, we have 8 uncertain parameters. In order to design the controller, the open-loop-plant is augmented with given weighting functions. The state space realization of the augmented open-loop plant is in the form

$$P(s) : \begin{bmatrix} z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A^\Delta & B_w & B_u^\Delta \\ \hline C_z^\Delta & 0 & D_{zu}^\Delta \\ C_y^\Delta & D_{yw} & D_{yu}^\Delta \end{array} \right] \begin{bmatrix} w \\ u \end{bmatrix} \tag{2}$$

where matrices with superscript Δ represents those which include uncertain parameters. The goal is to design a dynamic output feedback controller $K(s)$ which minimizes the $\mathcal{H}_2$ norm of the transfer function from disturbance to output while guaranteeing robust stability in the presence of parametric uncertainty. This problem can be formulated as the following optimization problem

$$K^* = \arg \min_{K \in \mathbf{K}} \|T_{zw}(s, K)\|_2 \tag{3}$$

where $\mathbf{K}$ is the set of all controllers which robustly stabilize the uncertain plant (2) and $T_{zw}(s, K)$ is the closed-loop transfer function

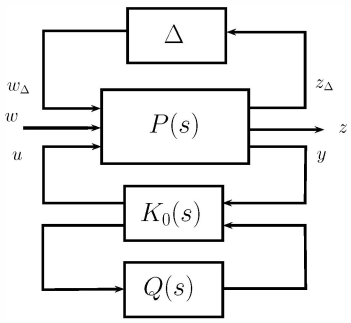

Fig. 1. Interconnection of the plant $P(s)$ with uncertainty block Δ, central controller $K(s)$ and $Q(s)$

from disturbance to output. The uncertain open-loop system is represented as a feedback connection of the extended plant with a block of structured uncertainties as depicted in Fig. 1.

III. ROBUST CONTROLLERS PARAMETERIZATION

In this section, we are concerned about the internal stabilization problem for the uncertain LTI system (2). More precisely, the objective is to find a controller $K_0(s)$ which makes the closed-loop system to satisfy the inequality

$$\|T_{z_\Delta w_\Delta}(s, K_0)\|_\infty < \gamma \tag{4}$$

where γ is a desired performance level and K_0 is called the central controller. Satisfaction of (4) guarantees robust stability for all $\Delta \in \mathbf{\Delta}_H$ where $\mathbf{\Delta}_H$ is defined as

$$\mathbf{\Delta}_H = \{\Delta \in \mathcal{RH}_\infty : \|\Delta\|_\infty \leq \frac{1}{\gamma}\}.$$

All controllers achieving internal stability and satisfying the norm bound (4) can be parameterized [2] by the set $\mathcal{Q}(\gamma) = \{Q(s) \in \mathcal{RH}_\infty : \|Q(s)\| \leq \gamma\}$. Therefore the set of all stabilizing controllers which achieve the norm bound (4) is defined as $K_Q(s) = \mathcal{F}_l(K_0(s), \mathcal{Q}(\gamma))$ and $\mathcal{F}_l$ denotes the standard lower fractional transformation.

IV. PERFORMANCE SPECIFICATIONS

In this section, we search though the space of all stabilizing controllers $K_Q(s)$ characterized by $\mathcal{Q}(\gamma)$ to find the optimal controller which solves (3). In particular, we desire to solve the following optimization problem:

$$Q^*(s) = \arg\min_{Q(s)} \eta$$
$$\text{subject to } \|T_{zw}(s, Q)\|_2 < \eta,\ Q(s) \in \mathcal{Q}(\gamma). \tag{5}$$

In the absence of the second constraint, we could easily convexify the problem by introducing nonlinear transformations and change of variables.

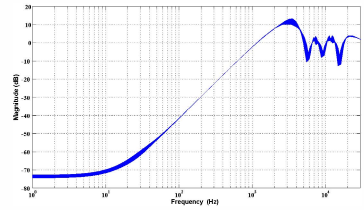

Fig. 2. The sensitivity transfer function for 500 randomly selected plants from the uncertainty set

However, by imposing the stability and norm bound constraints on $Q(s)$, which is playing the role of the controller here, we cannot recast the problem (5) into a convex problem. To prevent dealing with nonlinear matrix inequality (NMI), we have represented the problem as bilinear matrix inequality (BMI) for which there are efficient local solvers. The BMI formulation is omitted here in the interest of space. The resulting controller can be obtained as $K^*(s) = \mathcal{F}_l(K_0(s), Q^*(s))$.

V. SIMULATION STUDY

Simulations are carried on to examine the effectiveness of the designed controller. We used Quasi-Newton algorithm based on BFGS [3] to find an initial feasible point and then we used PENBMI solver [4] to find a (local) minimum of (5). Fig. 2 shows the sensitivity graph for 500 random samples extracted from the uncertainty set.

VI. CONCLUSION

The design is carried in three steps; in the first step, we design a controller based on μ-synthesis and H_∞ design to guarantee robust stability (without taking into account the performance channel). In the second step, the set of all stabilizing controllers is characterized by a stable and norm bounded transfer function $Q(s)$ and in the final step, the performance channel is considered and the original problem is reformulated as BMI optimization.

REFERENCES

[1] R. Tempo, G. Calafiore, and F. Dabbene, *Randomized Algorithms for Analysis and Control of Uncertain Systems*. Springer-Verlag, London, 2005.

[2] J. Doyle, K. Glover, P. Khargonekar, and B. Francis, "State-space solutions to standard H_2 and H_∞ control problems," *IEEE Transactions on Automatic Control*, vol. 34, no. 8, pp. 831–847, Aug. 1989.

[3] J. V. Burke, D. Henrion, A. S. Lewis, and M. L. Overton, "HIFOO-A MATLAB package for fixed-order controller design and H_∞ optimization," in *Fifth IFAC Symposium on Robust Control Design, Toulouse*, 2006.

[4] M. Kovara and M. Stingl, "PENNON: a code for convex nonlinear and semidefinite programming," *Optimization Methods and Software*, vol. 18, no. 3, pp. 317–333, 2003.