Design of Track Following Controller of Dual Actuated HDD Servo for 10 Tb/in^2 Magnetic Recording

Ehsan Keikha, Mohammadreza Chamanbaz, Abdullah Al-Mamun and Charanjit Singh Bhatia

***Abstract*—One of the obstacles to achieving high recording density in hard disk drives (HDDs) is servo performance limitation.As HDD is a mass produced commodity, the challenge is not only achieving high performance for nominal models but with all the uncertainties and variations in HDD models for thousands of HDDs in a batch of production line. This paper presents the implementation results of a novel technique based on the $\mathcal{H}_\infty$ probabilistic robust approach for the track-following controller of HDD servo systems. The objective is to design and implement an $\mathcal{H}_\infty$ dynamic output feedback controller that achieves robust stability in the presence of various parametric uncertainties. We consider uncertainty as a random variable with uniform probability distribution. Then a randomized algorithm based on gradient iteration is preformed to compute design parameters. The proposed controller is implemented in real-time on a commercially available drive and the results are compared with the conventional $\mathcal{H}_\infty$ approach. The results reveal that with a very small risk of violation of the cost function, the proposed method can achieve much better performance compared to the conventional robust methods.**

I. Introduction

In the recent decades, data storage industry has faced an ever increasing global demand for storage capacity. So far, hard disk drives have been the inimitable candidate to meet this demand due to its cost effectiveness and reliability. Recently, with the rapid progress of its competitor, the solid state drive, which is also gaining market share in storage capacity, higher performance is in demand for the HDD. To maintain the advantage HDD has at present, it is of vital importance to increase the areal density of HDD by 40% - 60% every year [1]. The recording density in commercially available HDDs still lies below 1 $Tbits/in^2$ but the next milestone is predicted to be 10 $Tbits/in^2$. Assuming bit aspect ratio (bit length : bit width) of 2:1, track density of about 2,200,000 tracks per inch (TPI) is required to achieve this predicted density, which can be translated into track pitch of 11.6 nm. The desired tracking error in positioning the read/write head above a track on the rotating disk while reading/writing data is less than 10% of the track pitch [1] . As a result, track miss-registration (TMR) is less than 1.16 nm for 10 $Tbits/in^2$ recording.

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Such high performance must be achieved in a robust manner because of the plant uncertainty in HDD dynamic systems. HDDs are normally produced in huge batches, although all of them have shared the same nominal properties, each drive has slightly different dynamic response. After servo control is embedded in disk drive systems, for each individual disk drive it is not feasible to fine-tune the controller parameters. Hence the same controller should stabilize and perform well on all these disk drives. Consequently the main challenge in designing HDD servo controller is to improve the nominal performance; meanwhile, the controller should be able to make the closed loop system robust against variations in dynamics. It is a known fact that high performance and sufficient robustness are conflicting technical requirements. To address this issue multi-variable optimal control design received considerable attention in HDD servo design. These methods can consider coupling dynamics and plant uncertainty systematically during the design process. Several control techniques such as Robust $\mathcal{H}_2$, Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ and μ-synthesis have been extensively studied for HDD servo control [2], [3], [4]. However the classical multi-objective controller design framework typically involves a large number of performance requirements. Deterministic optimization tends to make several requirements critical in a deterministic sense and imposes a number of theoretical limitations. Computational complexity is the first critical issue of the deterministic robustness paradigm. Various robust control problems have been proven to belong to the category of "intractable" problems,which generally denoted as "NP-hard" [5]. Generally, the design procedure of robust output-feedback controllers for an uncertain system, leads to bilinear (or rather bi-affine) matrix inequality terms (BMIs)[6], which are NP-hard in nature[7]. The computational complexity is even higher for the systems with parametric uncertainty. Guaranteed stability-bound estimations are often excessively conservative, and the resulting controller usually needs very high control effort.Available algorithms to solve such problems are typically based on the iterative methods which require large computation time, especially for higher-order plants. Several attempts have been made to find a practical solution to this design problem. For instance, some researchers have proposed to reduce the number of uncertainties by projecting them in a lower dimensional space using principal component analysis and nonlinear least-square [8]. Although this method reduces the computational complexity, it will also affect the robustness since the effect of noise in experimental data is not taken into consideration. Moreover, to overcome the conservatism, Conway et al. [8] proposed the approach of utilizing the parameter dependant Lyapunov function to design a single robust controller for parametric uncertain system which led to an improvement in the performance of HDD by introducing the $\mathcal{G}$ parameter which adds a greater degree of freedom. However, due to the limitations in the existing numerical solvers, this method is not suitable for real-world industrial applications and it can only solve problems with a very few uncertainties (only one in the cited paper). In addition to the complexity problem, conservativeness is also a challenge of the deterministic robust approach. It is well known that in cases where real parametric uncertainty enters affinely into the plant transfer function, it is possible to compute the robustness margin exactly. However, in real-world problems, we usually deal with non-linear uncertainty; for instance, each resonance mode in HDD contains the product term $\zeta_i\omega_i$ as well as the square term ω_i^2. In order to handle this problem in the classical robust paradigm, the non-linear uncertainty will be embedded into the affine structure by replacing the original set by larger one. In other words, multipliers and scaling variables are introduced to relax the problem [9] ,which are associated with an evident conservatism. Also in uncertain systems with dynamic uncertainty described in $M - \Delta$ form a similar situation arises. For instance, for systems affected by more than one full real block, the computation robustness margin (μ_D) can be generally performed only in a conservative way. In fact, in this cases, only upper and lower bounds of μ_D can be computed, but it may be difficult to estimate the degree of conservatism introduced.

The solution which has been introduced recently,is based on incorporating probabilistic concepts in robustness. In the new approach -the so called probabilistic robust design- uncertainty is considered as random variables and the probability of violation of performance function is ”estimated” rather than exactly computed. This estimation is carried by means of Monte-Carlo type of algorithms. The main feature of this approach is to apply convex optimization in the design parameter space and randomization in the uncertainty space. Therefore, thanks to randomization, the uncertainty set is treated as it is and no conservatism is introduced which tends to significant improvement in the performance of the closed loop system. Using probabilistic concept in robust control was first introduced by Stengel in 1980 [10] and later on it was used in [11] which deals with estimating the probability of instability using Monte-Carlo type of simulation. Randomized algorithm was further developed in [12], [13] where some explicit sample bounds have been derived based on which we can determine the probability of violation (or satisfaction) of a given cost function. Later on the idea of statistical leaning theory was introduced by Vidyasagar in his seminal paper [14]. Randomized algorithm have been employed to solve some control problems such as linear quadratic design [15] and solving uncertain LMI by means of gradient [16], ellipsoid [17] and cutting plane [18] iterations. Also some randomized algorithms are introduced based on zero sided Monte Carlo simulation which is known as Las Vegas algorithm [19]. Randomized algorithms are successfully used in a number of applications ranging from design of truss structure [20], UAVs [21], stability and robustness of communication networks [22] and page rank computation [23].

The main contribution of this paper is to partially overcome the above mentioned limitations regarding classical robust design. In particular, this approach is less computationally expensive while classical robust design suffers from considerable computational complexity while dealing with parametric uncertainty. Moreover, most results in classical robust design are applicable to polytopic uncertain systems; while, HDD is not a polytopic uncertain system and in order to change the uncertain system into a polytopic uncertain system some conservatism will be introduced.The reminder of the paper is organized as follow: in Section II,the experimental system identification of a hard disk drive is presented. The classical framework of the $\mathcal{H}_\infty$ is studied in Section III-A , which is followed by the procedure for designing the probabilistic controller in Section III-B, Section IV is dedicated to simulation results and the experimental results appear in Section V. Finally, some concluding remarks are given in Section VI.

II. System Identification of a Dual Stage HDD

In order to design controllers for any dual-stage actuation servo system, the plant models of both the voice coil motor (VCM) and secondary actuator (PZT) need to be obtained using the frequency domain system identification technique. Fig. 1 is an illustration of the experimental setup is used to determine the frequency response of both actuators. The setup consists of a Dynamic Signal Analyzers (DSA), a Laser Doppler Vibrometer (LDV), VCM and PZT power amplifier and spindle driver at 7200 RPM. The device under test (DUT) is a 3.5” HDD from Western Digital.

In the head positioning servomechanism of HDD, position feedback is obtained from the read-back signal produced by sensing special magnetic patterns written on the disks. Since the position error signal (PES) of the servo mechanism was not directly available, a LDV is used to measure the radial motion of read/write head slider . The DSA is used for generating excitation signals as well as measurement of response. The acquisition of output signals is carried out using a high-performance, high-accuracy analog I/O device (NI PXI 4461). This board has two digital to analog converters (DAC) and two analog to digital converters (ADC), though only one ADC is used. Each channel has its own sigma-delta converter with 24-bits of resolution and a maximum sampling rate of 100 KS/s. The excitation signal and the LDV output are fed to two channels of the DSA. The magnitude and phase of the LDV output are compared to the measured stimulus signal to calculate the FRF. The velocity of the HDD’s head is measured using the LDV (OFV 5001) with the LDV range set at 100nm/s/V during all experiments. The integration time and settling time are set as 10 cycles and also and the resolution

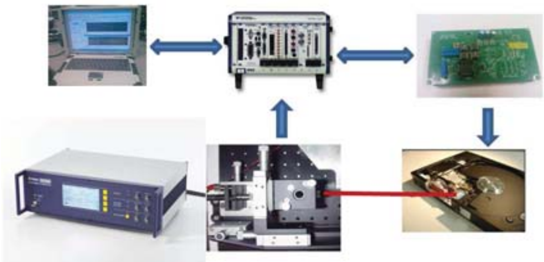

Fig. 1. The experimental setup block diagram

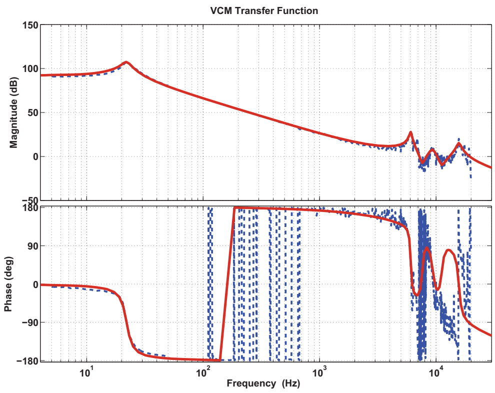

Fig. 2. Measured as well as identified frequency response for VCM actuator

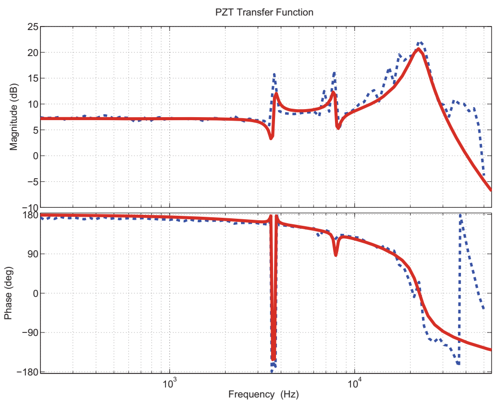

Fig. 3. Measured as well as identified frequency response for PZT actuator

for each sweep is set as 400 Points. While measuring the frequency response of the VCM, the input to the PZT is set to zero and vice versa.

Fig. 2 and Fig. 3 show the measured frequency response as well as the estimated models of VCM and PZT. The experimental data shows several distinct resonance modes. As a result, it is appropriate to describe the transfer function between each input-output pair as a summation of N modes, as follows. The transfer function of VCM and micro-actuator

TABLE I
IDENTIFIED MODAL PARAMETERS

n	ω_i (rad/sec)	ζ_i	A_i	M_{ω_i}	M_{ζ_i}	M_{A_i}
1	$2\pi \times 22$	0.08	7.6e8	5%	10%	5%
2	$2\pi \times 6063$	0.02	-1.5e9	5%	10%	5%
3	$2\pi \times 9358$	0.06	-1e9	5%	10%	5%
4	$2\pi \times 15693$	0.02	-4.5e9	5%	10%	5%

n	ω_{mi} (rad/sec)	ζ_{mi}	A_{mi}	$M_{\omega_{mi}}$	$M_{\zeta_{mi}}$	$M_{A_{mi}}$
1	$2\pi \times 3690$	0.02	-6e7	5%	15%	10%
2	$2\pi \times 7845$	0.02	2.4e8	5%	15%	10%
3	$2\pi \times 22360$	0.01	4.5e10	5%	15%	10%

are considered to be in the form of:

$$G_{VCM} = \sum_{i=1}^{4} \frac{A_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{1}$$

$$G_{MA} = \sum_{i=1}^{3} \frac{A_{mi}}{s^2 + 2\zeta_{mi} \omega_{mi} s + \omega_{mi}^2} \tag{2}$$

All the coefficients are uncertain values in the form of :

$$\omega_i = \bar{\omega_i}(1 + M_{\omega_i}\delta i) \prec 0 \quad , \zeta_i = \bar{\zeta_i}(1 + M_{\zeta_i}\delta i) \tag{3}$$

for each mode $\bar{\zeta_i}$ is the nominal damping ratio, $\bar{\omega_i}$ is the nominal natural frequency and $\bar{A_i}$ is the nominal modal constant. This form requires only three parameters per mode: ζ_i is damping ratio, ω_i is natural frequency and A_i is modal constant and finally δi s are perturbations which are norm bounded. The same notations have been used for PZT formulation.All parameters in the model give an insight into the physical plant, and the nominal parameters for the examined disk drive are given in the Table I.

III. Controller Design

In this section, the procedure for designing controller is presented. A couple of steps should be followed for designing the controller; in particular, the nominal controller is first designed and then it is robustified against parametric uncertainties using a randomized algorithm based on gradient iteration. In the following sections design steps are describe in detail.

A. Classical $\mathcal{H}_\infty$ Controller Design

In this section, a classical design of a $\mathcal{H}_\infty$ controller is presented. There are two approaches for such a design which are based on the Riccati equation and linear matrix inequality (LMI) [24]; however, since the randomized algorithm is able to find the probabilistic feasible set of uncertain LMI, then the design based on LMI is given here. In the design based on LMI the general approach is to employ bounded real lemma (for $\mathcal{H}_\infty$ design) and then introduce non-linear transformation and slack variables in order to change the problem into linear (or rather affine) form. As a result of bounded real lemma, A is stable and the $\mathcal{H}_\infty$ norm of closed loop is smaller than γ if and only if there exists a symmetric P which satisfies the bellow conditions

$$\begin{pmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{pmatrix} \prec 0 \quad , P \succ 0 \qquad (4)$$

then by replacing matrixes A, B, C and D with the closed loop matrixes (in terms of plant and control parameters) and introducing slack variables and non-linear transformations, the problem is reformulated as LMI for which there are a lot of efficient numerical algorithms and solvers.

B. Randomized Algorithm for Probabilistic Design

In this section, a randomized algorithm based on gradient iteration is presented. The algorithm is able to solve uncertain LMIs in the probabilistic sense. In order to design the controller, we first need to define a scalar function which is useful for evaluating the rate of violation. In this paper, the norm of projection of matrix inequality (4) into the cone of symmetric positive semi-definite matrixes is used as the violation function. This is a non-negative and convex function and it is positive if and only if the matrix inequality is violated for a particular design parameter and randomly generated scenario of the uncertainty set. The algorithm proceeds as follows:

1) *Initialization*: Set iteration counters to zero choose an initial candidate.
2) *Oracle*: Estimate the probability of violation of matrix inequality (4). If it satisfies the requirements, terminate the iteration , candidate solution is probabilistic feasible solution if not proceed to the next step.
3) *Update rule*: Update design parameters based on gradient iteration [16].
4) *Outer iteration*: Go to step two.

The convergence of the given algorithm can be proven provided that "strict feasibility" and non-zero probability of detecting unfeasibility" assumptions hold. In step one, any initial candidate can be chosen; however, the case of initial candidate will affect the convergence time. Then it is better to put solution to the nominal case as initial candidate. Step two is responsible for "estimating" the violation probability which is based on Monte-Carlo simulation and the sample size is chosen based on the well known "log over log" bound [25]. Generally speaking, in this step we draw a number of (defined by log over log bound) randomly selected uncertain plants from the uncertainty set and we check if the violation function is zero for "all" of them; if so, the candidate solution is probabilistically robust one and if not we update the design parameter based on:

$$\theta_{k+1} = \left[\theta_k - \vartheta_k \partial_\theta \left\{\tau(\Delta_i, \theta_k)\right\}\right]_\Theta \qquad (5)$$

in which θ is design parameter, ϑ determines the step size and ∂_θ is gradient with respect to θ and τ is violation function.

As stated earlier one of the main challenges to design a robust controller for HDD servo systems is the high dimension of uncertainty set in both parametric and dynamic form. Consequently, to synthesis the controller the probabilistic approach is suggested rather than use of a classical $M - \mathbf{\Delta}$ framework.

TABLE II
IDENTIFIED MODAL PARAMETERS

	Probabilistic H_∞	Conventional H_∞
$10\% setteling time$	$0.542 msec$	$0.53 msec$
over shoot	22.3%	19.5%
crossover frequency	$2.89 Khz$	$3.49 Khz$
gain margin	$11.2 dB$	$11.4 dB$
phase margin	$61.4 degree$	$63.6 degree$
sensitivity peak	$3.85 dB$	$4.6 dB$
Control signal level		
$\|U_{vcm}\|_\infty$	$0.0822V$	0.0862 V
$\|U_{pzt}\|_\infty$	$0.3208V$	0.3193 V

IV. SIMULATION RESULT

To evaluate the controller performance, some simulations are carried out for the closed loop system. The control objective is to design an $\mathcal{H}_\infty$ controller that *robustly* stabilizes the closed loop plant in the presence of uncertain parameters.

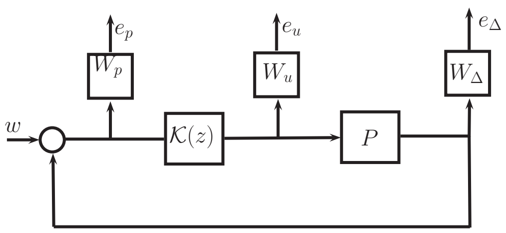

Fig. 4. Generalized Open Loop Plant

The block diagram of the extended open loop system is shown in Fig. 4. P denotes the nominal plant, while W_u, W_n and W_p are the performance weighting functions .

In order to design the controller, the generalized plant should be represented in the state space form. Then the algorithm which is presented in Section III-B is carried using Matlab [26]. We assumed a uniform probability distribution due to its worst case nature, while sampling the uncertainty space. In order to reduce the number of outer iterations in the algorithm, we solved the $\mathcal{H}_\infty$ problem for nominal plant using YALMIP [27] and the results are given to the randomized algorithm as the initial value. After a number of iterations which basically depend on initial probabilistic levels, algorithm comes up with the design parameters that makes the closed-loop plant robustly stable (in a probabilistic way).

To further validate our design, a posteriori analysis is carried out for the designed controller. To do this, 50 random plants are chosen from the set of uncertainty that appears in Table I, then we simulate closed loop response for each of these plants with the designed controller. Fig. 5 shows the closed loop sensitivity plot for the designed controller and Fig. 6 demonstrates the same plot for the case where $\mathcal{H}_\infty$ controller is designed using YALMIP without considering any

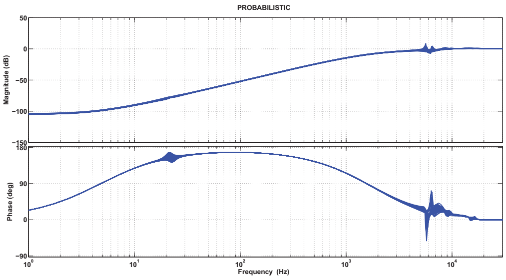

Fig. 5. Closed loop sensitivity plot with controller designed using probabilistic framework for 100 random samples of $\Delta \in \mathbf{\Delta}$

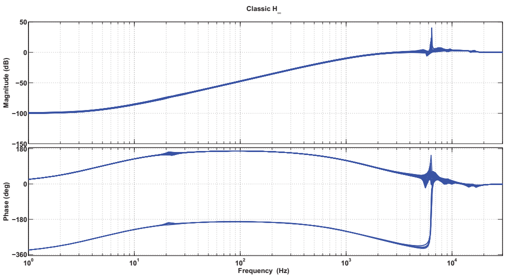

Fig. 6. Closed loop sensitivity plot with $\mathcal{H}_\infty$ controller designed using YALMIP for 100 random samples of $\Delta \in \mathbf{\Delta}$

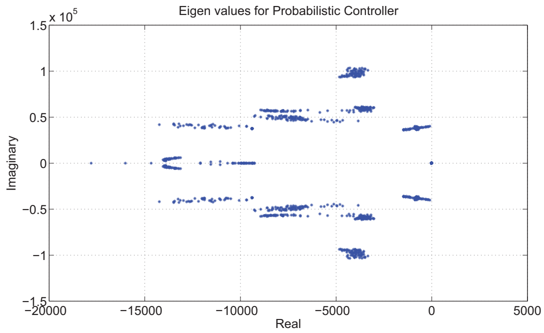

Fig. 7. Closed loop eigenvalues plot with controller designed using probabilistic framework for 100 random samples of $\Delta \in \mathbf{\Delta}$

Fig. 8. Closed loop eigenvalues plot with $\mathcal{H}_\infty$ controller designed using YALMIP for 100 random samples of $\Delta \in \mathbf{\Delta}$

uncertainty. As it is clear from the plots, the probabilistic

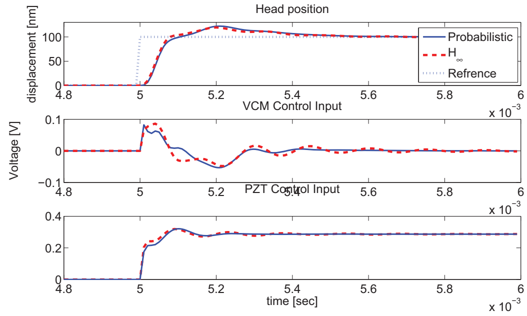

Fig. 9. Simulation of head position and control signals for $\mathcal{H}_\infty$ and probabilistic methods

controller degrades much less over the uncertain parameter set. In addition, Fig.7 and Fig. 8 shown the closed loop eigenvalues of the system. It is clear that the $\mathcal{H}_\infty$ design has become unstable for a number of realizations of in the uncertainty set since the closed loop eigenvalues have shifted to the right half plane. However, the probabilistic approach remains stable for all cases. This robustness is archived with minimum degradation in the nominal performance. Table II compares the performance measures of both controller. We observe that although the cross-over frequency of the $\mathcal{H}_\infty$ is higher, the probabilistic controller shows a better rejection for high frequency. Other measures are nearly the same. The step function is another common performance evaluation as an input representing either a short-term seek or high frequency disturbance. Fig. 9 shows the simulation response of a 100 nm step for both controllers, the control signals for both VCM and PZT. It is clearly evident from the simulation results that though both controllers preform nearly the same for the nominal plant, there is a distinct superiority of the probabilistic controller over $\mathcal{H}_\infty$ when robust performance is considered.

V. Control Implementation

The validity of the proposed controller is also verified by experiments. The implementation results are carried out at a sampling frequency of 20 kHz. It should be noted that the HDD used in this experiment is working at the nominal speed of 7200 RPM, and is placed on a vibration-free table. The displacement of the Read/Write head is measured by an LDV, and the real-time control is implemented using a dSpace package. A 50 Hz square wave is used as the reference signal. Closed loop response to this reference of the $\mathcal{H}_\infty$ and probabilistic controller are shown in Fig. 10 and Fig. 11 respectively. Each rise and fall in the reference signal is considered as a step trigger. Small differences are observed between experimental results and simulations, is due to the the disturbances such as repeatable runout at frequency of 120 Hz as well as the resonant mode of 3.6 kHz and 6 kHz of the actuators.

VI. Conclusion

In this paper, we have demonstrated design and implementation of a probabilistic robust $\boldsymbol{H}_\infty$ controller for a dual stage

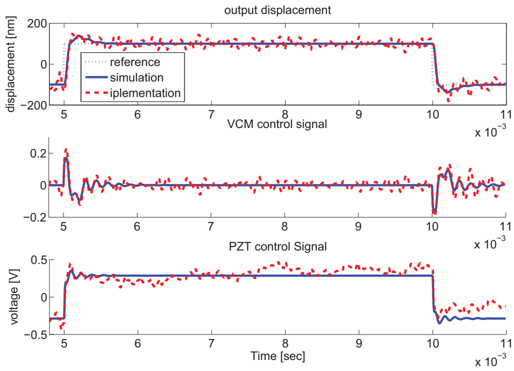

Fig. 10. Implementation of head position and control signals for $\mathcal{H}_\infty$ method

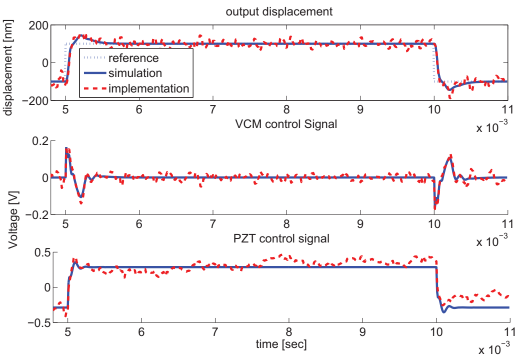

Fig. 11. Implementation of head position and control signals for probabilistic method

HDD. Such a controller benefits from the sequential technique based on gradient iteration. As can be seen from the simulation results, the parametric variation in the plant dynamic may tend to be unstable for the classical $\boldsymbol{H}_\infty$ controller while the presented controller remains robust in terms of performance and stability . Robustness and performance are two contradicting requirements; however, it can be seen that the given controller has a fairly good performance compared to the nominal controller. The experimental condition is as close as possible to normal operating condition of HDD and the experimental results are quite similar to simulation. The slight differences are due to the high frequency unmodeled dynamics and repeatable disturbances.

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