$\mathcal{H}_\infty$ Probabilistic Robust Control of Hard Disk Drive

Mohammadreza Chamanbaz
Data Storage Institute, Singapore and
Electrical &
Computer Engineering Department
National University of Singapore
Email: Mohammad_C@dsi.a-star.edu.sg

Ehsan Keikha
Electrical &
Computer Engineering Department
National University of Singapore
Email: Ehsan.keikha@nus.edu.sg

Venkatakrishnan Venkataramanan
Data Storage Institute, Singapore
Email: Venka_V@dsi.a-star.edu.sg

Abdullah Al Mamun
Electrical &
Computer Engineering Department
National University of Singapore
Email: Eleaam@nus.edu.sg

Wang Qing-Guo
Electrical &
Computer Engineering Department
National University of Singapore
Email: Elewqg@nus.edu.sg

Thomas Liew
Data Storage Institute, Singapore and
Electrical &
Computer Engineering Department
National University of Singapore
Email: TOMLIEW@dsi.a-star.edu.sg

***Abstract*—This paper presents a $\mathcal{H}_\infty$ probabilistic robust track following output feedback controller for Hard Disk Drive (HDD) servo systems. Such a controller benefits from the sequential approximation method based on gradient iteration. In probabilistic framework, randomization is used to handle uncertainty and convex optimization to find the design parameters. In this approach, uncertainty is considered as random variable; hence, due to this stochastic nature, the probability of performance function can be estimated. Simulation results reveal that by allowing a very small violation of cost function, we can achieve much better performance compared to classical deterministic approach.**

I. Introduction

In recent five decades, data storage industry has faced an ever increasing global demand for data storage capacity. Currently HDD uses combination of classic control techniques such as PID control, lead lag compensators and notch filters [1]. On the other hand, due to the higher performance demand, the classical control methods no longer can be used in higher capacity HDDs. One of the main challenges is to improve the performance of the system as much as possible; meanwhile the controller has to be robust against variations in dynamics. However, high performance and sufficient robustness are conflicting requirements; to address this issues, robust control design based on multi-objective optimization method received considerable attention in HDD servo design. Several control techniques such as Robust $\mathcal{H}_2$, Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ and μ-synthesis have been extensively studied for HDD servo control [2], [3]. However, classical multi-objective controller design framework suffers from a number of theoretical limitations due to its deterministic nature. These limitations can be generally divided into two main groups, namely computational complexity and the issue of conservatism. Various robust control problems have been proved to belong to the category of "intractable" problems, which generally denoted as "NP-hard" [4]. In general the design of robust output-feedback controllers lead to bilinear (or rather bi-affine) matrix inequality terms (BMIs) [5], which are NP-hard in nature [6]. Available algorithms to solve such problems are typically based on the iterative methods which are computationally expensive, specially for higher-order plants. Several attempts have been carried to propose a practical design method; for instance, in [7] authors have proposed to reduce the number of uncertainties by projecting them in a lower dimensional space using principal component analysis and nonlinear least-square. Although this method reduces the computational complexity, it will also affect the robustness since the effect of noise in experimental data is not taken into consideration. Also, to overcome the conservatism, Conway et al. in [8] proposed the approach of utilizing parameter dependant Lyapunov function to design single robust controller for polytopic parameter uncertain system which led to improvement in performance of HDD by introducing $\mathcal{G}$ parameter which adds more degree of freedom. However, due to the limitations in the existing numerical solvers, this method is just applicable to problems of modest size with very few (only one in the paper) number of uncertainties. In addition to the complexity problem, conservativeness is also a challenge for the deterministic robust approach. It is well known that in cases where real parametric uncertainty enters affinely into plant transfer function, it is possible to compute the robustness margin exactly. However, in real world problems, we usually deal with non-linear uncertainty; for instance, each resonance mode in HDD contains the product term $\zeta_i\omega_i$ as well as the square term ω_i^2. In order to handle this problem in classical robust paradigm, the non-linear uncertainty will be embedded into affine structure by replacing the original set by larger one. In other words, multipliers and scaling variables are introduced to relax the problem [9] ;which are associated with an evident

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conservatism.

Hence, a fairly new approach, the so called *probabilistic robust control*, was proposed to overcome above mentioned problems. In the new approach, the robustness is determined in probabilistic sense rather than the classical deterministic one. We accept a small risk that objective function being violated for a set of uncertainty having small probability measure [10]. Since performance and robustness are contrary specifications, the probabilistic approach seem to be a useful way to tackle limitations regarding the deterministic worst case approach. It turns out that if we allow a small violation of cost function, say 10^{-5}, the performance of resulting controller will be improved significantly. The main feature in this line of research is to use probabilistic concept rather than classical deterministic one. Early appearance of this approach goes back to 1980s in the field of flight control by Stengel [11]. Some papers have been published in the field of flight control during 1980s and early 1990s which mostly deal with analysis problems based on Monte Carlo simulation. Later on some results based on explicit sample size bound [12], [13] , statistical learning theory by Vidyasagar in his seminal paper [14] and Las Vegas RAs [15] were presented into literature that deals with both synthesis and analysis problems. Mixed probabilistic deterministic approach also is presented [16] to overcome the problems regarding probabilistic approach and to assure stability of closed loop system. Finally, RAs are used in a number of control applications including design of truss structure [17], UAVs [18] and stability and robustness of high speed communication networks [19]. For a comprehensive study, interested readers are referred to the [20], [21] as well as survey papers by G. Calafiore [10], [22]. The main contribution of this paper is to design a probabilistic $\mathcal{H}_\infty$ controller for a plant with uncertain parameters. This approach is benefited from nice properties of $\mathcal{H}_\infty$ control synthesize with considerably less conservativeness. In addition, this method is not computationally complex, while, when it comes to parametric uncertainty, classical deterministic schemes suffer from considerable computational complexity. The reminder of the paper is organized as follows: in Section II-A, a $\mathcal{H}_\infty$ classical controller is designed , the procedure for designing the probabilistic controller is given in Section II-B, we introduce the generalized plant of HDD which consists of nominal model and uncertain parameters in Section III, Section IV is dedicated to simulation results. Finally, some concluding remarks are given in Section V.

Notation

For a square matrix X,$X \succ 0$ ($X \succeq 0$) means X is symmetric and positive definite (semidefinite). I_n denotes $n \times n$ identity matrix. The symbol $\mathcal{RH}_\infty$ denotes the space of stable rational transfer matrices. The $\mathcal{H}_\infty$ norm of a transfer matrix $Q(s) \in \mathcal{RH}_\infty$ is denoted as $\|Q\|_\infty$ and $\|X\|_F$ denotes Frobenius norm of X. The symbol $\lceil x \rceil$ is the minimum integer greater or equal to $x \in \mathbb{R}$. The symbol $\bullet$ in LMIs denotes entries that come from symmetry. Boldface capital letters are used for showing variable matrixes. Also, v_i denotes the ith element of vector v. The projection of a real matrix A onto the closed set S is defined as

$$[A]_S \doteq arg \min_{S \geq 0} \|A - S\|_F \tag{1}$$

In the special case where S is the cone of symmetric positive definite matrices, we use

$$[A]_+ \doteq arg \min_{X \in S} \|A - X\|_F \tag{2}$$

To indicate that the matrixes (A, B, C, D) are a state space realization of a transfer matrix $Q(s)$, we use the standard notation

$$Q(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(sI - A)^{-1}B + D$$

II. Controller Design

In this section, we discuss the procedure for designing probabilistic controller. In order to do so, first we need to formulate the problem into classical $\mathcal{H}_\infty$ framework then; $\mathcal{H}_\infty$ design is reformulated into LMI formation and finally, the algorithm to solve uncertain LMI based on gradient iteration is presented.

A. $\mathcal{H}_\infty$ Controller Design

$\mathcal{H}_\infty$ control is a well known control design methodology for synthesizing control systems that achieves robust performance or stability. One of the most popular methods for designing $\mathcal{H}_\infty$ controller is based on LMI [23]. The reason behind this popularity is that solving LMI is a convex optimization problem and there are several numerically efficient algorithms and softwares to solve LMI problems. The main purpose of this section is to give an overview of the $\mathcal{H}_\infty$ design technique based on LMI. LTI model of the plant can be described in state space form as

$$P: \quad \begin{bmatrix} z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_{ew} & D_{ew} & D_{eu} \\ C_y & D_{yw} & 0 \end{array}\right] \begin{bmatrix} w \\ u \end{bmatrix} \tag{3}$$

where $u \in R^{n_u}$ is the control input vector, w is a vector of exogenous signals such as disturbance, reference signal and sensor noise, $y \in R^{n_y}$ and z are measurement and output vectors respectively. The goal is to design a dynamical output feedback controller in the form of

$$u = K(s)y = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & 0 \end{array}\right] y \tag{4}$$

that stabilizes the plant and minimizes the infinity norm of closed-loop transfer function from exogenous signal w to output z, assuming (A, B_u) being stabilizable and (C_y, A) detectable. With the plant P and controller K, we can define the closed-loop system as

$$G(s) = \mathcal{F}_l(P(s), K(s)) = \left[\begin{array}{cc|c} A & B_uC_k & B_w \\ B_kC_y & A_k & B_kD_{yw} \\ \hline C_e & D_{eu}C_k & D_{ew} \end{array}\right] \tag{5}$$

where $\mathcal{F}_l$ denotes lower fractional transformation. $\mathcal{H}_\infty$ optimal control design objective is

$$\underset{k\in\mathcal{K}}{minimize}\, \|\mathcal{F}_\ell(P(s),K(s)\|_\infty \quad (6)$$

where $\mathcal{K}$ is the set of all stabilizing controllers. This problem can be reformulated in the form of LMI [23]. Then, alternatively instead of solving (6), we can solve the following optimization subject to a set of LMI's

$$minimize \quad \gamma \quad subject\,to \begin{cases} \begin{bmatrix} \mathbf{X} & I \\ I & \mathbf{Y} \end{bmatrix} \succ 0 \\ \begin{bmatrix} A\mathbf{X}+\mathbf{X}A^T+B_u\hat{\mathbf{C}}+(B_u\hat{\mathbf{C}})^T & \bullet & \bullet & \bullet \\ \hat{\mathbf{A}}+A^T & A^T\mathbf{Y}+\mathbf{Y}A+\hat{\mathbf{B}}C_y+(\hat{\mathbf{B}}C_y)^T & \bullet & \bullet \\ B_w^T & B_w^T\mathbf{Y}+D_{yw}^T\hat{\mathbf{B}}^T & -\gamma I & \bullet \\ C_e\mathbf{X}+D_{eu}\hat{\mathbf{C}} & C_e & D_{ew} & -\gamma I \end{bmatrix} \prec 0 \end{cases} \quad (7)$$

where $(\mathbf{X},\mathbf{Y},\hat{\mathbf{A}},\hat{\mathbf{B}}\ and\ \hat{\mathbf{C}})$ are decision variables. For controller reconstruction, first we need to compute M and N having the same block structure as $\mathbf{X}$ and $\mathbf{Y}$ and satisfying

$$NM^T = I - \mathbf{XY} \quad (8)$$

The controller parameters can be computed by solving the following equations for A_k, B_k and C_k

$$\begin{cases} \hat{\mathbf{A}} = NA_kM^T + NB_kC_y\mathbf{X} + \mathbf{Y}B_uC_kM^T + \mathbf{Y}A\mathbf{X} \\ \hat{\mathbf{B}} = NB_k \\ \hat{\mathbf{C}} = C_kM^T \end{cases} \quad (9)$$

A_k, B_k, C_k gives stable closed loop and $\|\mathcal{F}_l(P(s),K(s))\|_\infty \le \sqrt{\gamma}$. We will use the procedure discussed in this section to design the probabilistic controller in the next section. It should be noted that we didn't consider any uncertainty while designing the controller in this section.

B. Probabilistic Controller

The main idea behind the probabilistic framework is to use randomization to handle uncertainty and convex optimization to compute the design parameters. In this approach, we assume that uncertainty is a random variable Δ with probability density function (pdf) $f_\Delta(\mathbf{\Delta})$ and support $B_\mathbb{D}(\rho)$ where $B_\mathbb{D}(\rho)$ is uncertainty ball with radius ρ. Then our aim is to minimize the objective function (6) in probabilistic way; which mathematically can be expressed as

$$Pr\left\{J(\Delta,\theta) \le \sqrt{\gamma}\right\} \ge p^* \quad (10)$$

Where $J = \|\mathcal{F}_\ell(P(s),K(s)\|_\infty$, p^* is the given (high) probability level and $\theta \in \Theta$ is the vector of design parameters. It turns out that this problem is extremely difficult to solve exactly; since it requires the computation of multi-dimension integrals. However, we can *estimate* the performance probability using randomized algorithms. Mainly, there are three different methodologies to handle this problem. The first approach is based on Vapnik-Chervonenkis theory of learning [14], which tries to design a controller such that the average performance (with respect to Δ) is minimized; in this approach, randomization is carried in both uncertainty space and stabilizing controllers space. The second paradigm is based on scenario approach [24]. In this approach, a prespecified number of randomly selected independent scenarios are chosen and optimization is solved for this finite number of samples instead of infinite one. It should be noted that in scenario approach, we can handle optimization problems while sequential algorithms can just deal with feasibility problems. Finally, the third method is sequential approximation method based on gradient [25], ellipsoid [26] and cutting plane [27] iterations. In this paper, considering the nature of the problem, we use the sequential approximation method based on gradient iteration. In order to solve the problem (10), a particular function $\tau(\Delta,\theta)$, related to the objective function (6), is introduced. Essentially, the function $\tau(\Delta,\theta)$ measures the level of violation of performance function and is called *performance violation function*. The following properties hold for performance violation function:

1) $\tau(\Delta,\theta) \ge 0$ or all $\Delta \in B_\mathbb{D}(\rho)$ and $\theta \in \Theta$.
2) $\tau(\Delta,\theta) = 0$ if and only if $J(\Delta,\theta) \le \sqrt{\gamma}$.

As discussed in Section II-A, the optimization problem is a minimization subject to constrains in terms of LMIs (7). Since γ can be computed a priori for nominal case, then we can change the optimization problem into feasibility one for this special case. Then we need to find the probabilistically feasible solution of uncertain LMIs. We can easily express both LMIs (7) into a single LMI. By calling the first LMI in (7), $f_1(\theta,\Delta)$ and the second one $f_2(\theta,\Delta)$, then we can combine both LMIs into a single LMI [28] as follow:

$$F(\theta,\Delta) = diag(-f_1(\theta,\Delta), f_2(\theta,\Delta)) \prec 0 \quad (11)$$

Then the performance violation function can be expressed as

$$\tau(\Delta,\theta) = \left\|[F(\theta,\Delta)]_+\right\|_F \quad (12)$$

The aim is to minimize the performance violation function (12). The flowchart shown in Fig.1 proposes a randomized algorithm to find the probabilistic robust feasible solution to (11).

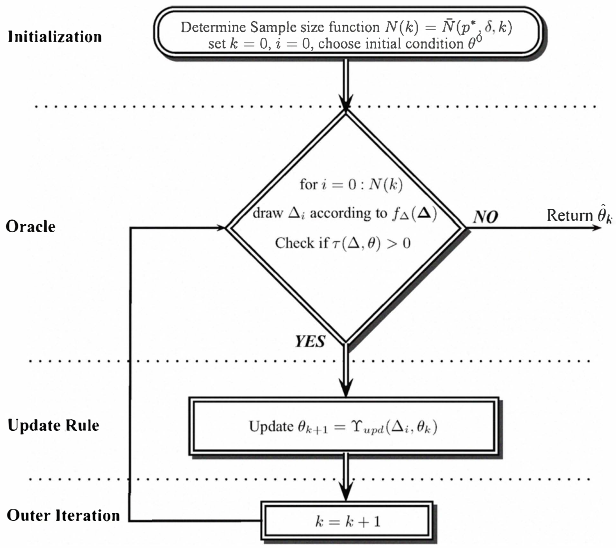

Fig. 1. Controller Design Flow Chart

It can be shown that [29] if the sample size function $N(k)$ is chosen as

$$N(k) = \overline{N}_{ss}(p^*, \delta, k) \doteq \left\lceil \frac{\log \frac{\pi^2 (k+1)^2}{6\delta}}{\log \frac{1}{p^*}} \right\rceil \tag{13}$$

where δ and p^* are given probability levels. Then, if the algorithm has successful exit, with probability *at least* $1-\delta$ the returned solution is probabilistically robust feasible solution to (11). Here we describe different parts of the randomized algorithm; first, we initialize the algorithm by choosing sample bound based on (13) and any initial condition. In "Oracle" part, the performance violation function is checked for $N(k)$ number of independent identically distributed (iid) samples and if it satisfies for all of the samples, the parameter $\hat{\theta}_k$ is returned as probabilistic robust feasible solution; otherwise, θ_k is updated according to

$$\theta_{k+1} = \Upsilon_{upd}(\Delta_i, \theta_k)$$

and this iteration is carried in finite number of iterations to find the solution. The following theorem [25] proposes an update rule with guaranteed convergence. In stating the next theorem, we assume that "strict feasibility" and "nonzero probability of detecting unfeasibility" assumptions hold. Interested readers may refer to [21].

Theorem 1. *For any initial condition θ_0, the following update rule finds the robustly feasible solution of (11) in finite number of iterations with probability one.*

$$\theta_{k+1} = \Upsilon_{upd}(\Delta_i, \theta_k) = [\theta_k - \vartheta_k \partial_\theta \{\tau(\Delta_i, \theta_k)\}]_\Theta \tag{14}$$

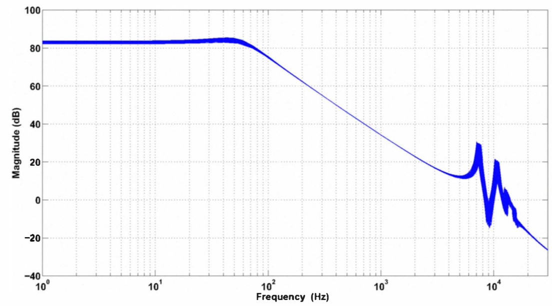

Fig. 2. VCM Frequency Response for 100 Randomly Selected Plants

Where ∂_θ is the subgradient with respect to θ, $[\cdot]_\Theta$ is the projection defined in (1) and

$$\vartheta_k = \frac{\tau(\Delta_i, \theta_k) + r \, \|\partial_\theta \{\tau(\Delta_i, \theta_k)\}\|_F}{\|\partial_\theta \{\tau(\Delta_i, \theta_k)\}\|_F^2} \tag{15}$$

and $r > 0$ is the radius of assumed feasible ball.

III. Generalized Plant

In order to verify the designed controller in the Section II and compare the results with the deterministic robust controller, the problem is solved for the case of HDD servo system. We have considered a single stage HDD; the plant model has been used in the literature [8], which consists of a rigid body and six resonance modes:

$$G_{VCM} = \sum_{i=1}^{7} \frac{A_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{16}$$

for each mode ζ_i is damping ratio, ω_i is natural frequency and A_i is modal constant. The parameters ζ_i and ω_i are assumed to be uncertain and their variation range is shown in Table.I. The frequency response of the uncertain VCM model is shown in Fig.2 for 100 randomly selected plants. Since we are going to make a comparison between probabilistic robust approach and deterministic framework, we need to reformulate the plant into the classical $M - \mathbf{\Delta}$ framework in order to design the deterministic robust controller. Then, the parametric uncertainty is expressed in multiplicative form as

$$\zeta_i \omega_i = \overline{\zeta}_i \overline{\omega}_i (1 + 0.15\eta_{ci}) \quad \omega_i^2 = \overline{\omega}_i^2 (1 + 0.1\eta_{di}) \quad i = 2 \ldots 7$$

where parameters with overbar e.g. $\overline{\omega}_i$, denotes the nominal values and η_{*i} are perturbations which are norm bounded. It is clear that, since transfer function coefficients must depend affinely on uncertain parameters in order to be able to solve the problem in classical robust paradigm, some conservatism is introduced while defining $\zeta_i \omega_i$.

This is one of the examples that shows the conservativeness of deterministic approach, however, in probabilistic paradigm, since we don't need to reformulate the system into classical $M - \mathbf{\Delta}$ framework, uncertain parameters are treated as they are. This is one of the superiorities of probabilistic framework over the classical deterministic one. The block diagram of the

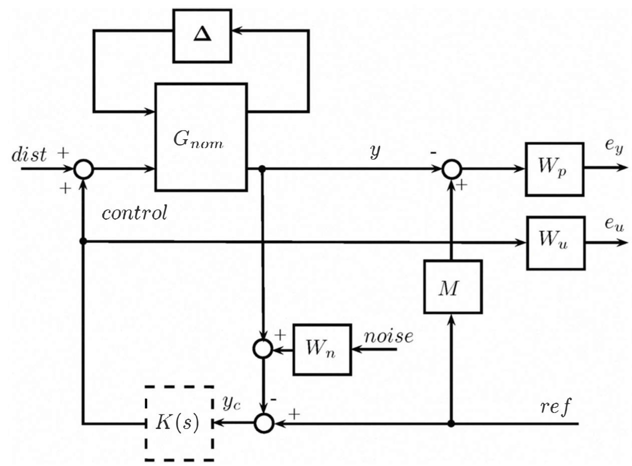

Fig. 3. Generalized Open Loop Plant

TABLE I
PARAMETER VARIATIONS IN THE MODEL

ζ_i	ω_i
$\pm 10\%$	$\pm 5\%$

extended open loop system is shown in Fig.3. G_{nom} denotes the nominal plant, M is ideal model of performance, in which the desired closed-loop specifications are defined, W_u, W_n and W_p are the performance weighting functions and finally, $\mathbf{\Delta}$ is the norm bounded perturbation.

IV. SIMULATION

Some simulations are carried to see the effectiveness of the proposed controller. The control objective is to design a $\mathcal{H}_\infty$ controller that *robustly* stabilizes the closed loop plant in presence of uncertain parameters. In classical robust approach, in cases where uncertainty is unstructured (e.g. high frequency unmodeled dynamics), it can be represented in the form of linear fractional transformation (LFT) and thanks to small gain theorem, the controller objective can be reformulated into controller design in absence of uncertainty. However, for structured uncertainty (such as parametric uncertainty in this case) following the same approach is over-conservative. We can reduce the conservativeness using μ-synthesis which consists of iteration procedure (known as **D-K** iteration) where at each iteration two convex optimizations are solved [30]. However, this procedure is not guaranteed to converge to even a "local" optimum. Then, considering all this pessimistic results, one of the novelties in probabilistic controller design is that by accepting a very small risk, the $\mathcal{H}_\infty$ controller is designed which robustly stabilizes the closed-loop plant. In order to design the controller, the generalized plant should be represented in the state space form. Then the algorithm which is presented in Fig.1 is implemented using Matlab [31]. We assumed uniform probability distribution, due to its worst case nature, while sampling the uncertainty space. In order to reduce the number of outer iterations in the algorithm, we solved the $\mathcal{H}_\infty$ problem for nominal plant using YALMIP

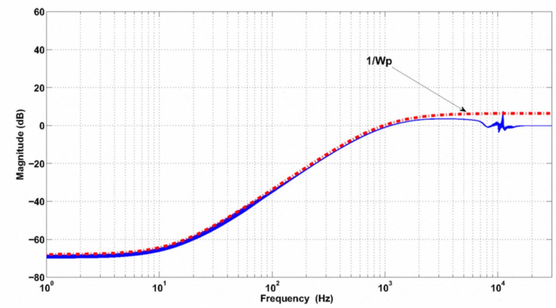

Fig. 4. Closed loop sensitivity plot with controller designed using probabilistic framework for 100 random samples of $\Delta \in \mathbf{\Delta}$

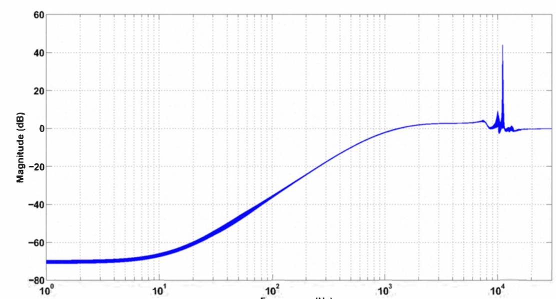

Fig. 5. Closed loop sensitivity plot with $\mathcal{H}_\infty$ controller designed using YALMIP for 100 random samples of $\Delta \in \mathbf{\Delta}$

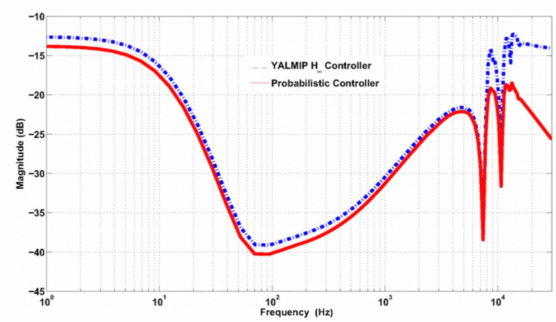

Fig. 6. Comparison between $\mathcal{H}_\infty$ controller designed using YALMIP and proposed probabilistic controller

[32] and the results are given to the randomized algorithm (Fig.1) as the initial value. After a number of iterations which basically depends on probabilistic levels p^*, δ (0.9999 and 10^{-6} respectively) and r, algorithm comes up with the design parameters that make the closed-loop plant robustly stable (in probabilistic way).

We tried to design a classical robust output feedback controller based on μ-synthesis, however, after including parametric uncertainties in the model the **D-K** iteration doesn't converge. Which means that classical robust schemes can only handle a few number of uncertainties (e.g. one or two) while in probabilistic approach, we are designing controller considering 12 parametric uncertainties. To further validate our design a posteriori analysis is carried for the designed controller; to do so, 100 random plants are chosen, then we closed the loop for each of these plants. Fig.4 shows the closed loop sensitivity plot for the designed controller as well as the inverse of performance weighting function (W_p) and Fig.5 demonstrate the same plot for the case where $\mathcal{H}_\infty$ controller is designed using YALMIP without considering any uncertainty. As it is clear from the plots, the probabilistic controller degrades much less over the uncertain parameter set. And finally, both controllers transfer function are shown in Fig.6 for comparison.

V. Conclusion

In this paper, we presented a probabilistic robust controller for track following of HDD servo systems. Objective is to design a $\mathcal{H}_\infty$ output feedback controller that achieves robust stability in presence of various parametric uncertainties. Uncertainty is considered as random variable with uniform probability distribution; then a randomized algorithm based on gradient iteration is performed to find design parameters. It should be noted that, no conservativeness is introduced in designing as well as modeling procedure. To the best of our knowledge, the best classical robust controller which can handle parametric uncertainties is robust $\mathcal{H}_2$ controller [2], however, this approach suffers from computational complexity. The computational complexity of robust $\mathcal{H}_2$ design increases exponentially with the number of uncertainties; then it can handle only a few parametric uncertainties [2]. In designed probabilistic controller, we considered 12 parametric uncertainties; some posteriori analysis was performed to verify the designed controller which clearly shows significant improvement compared to classical robust paradigm.

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