Preprints of the 7th IFAC Symposium on Robust Control Design
The International Federation of Automatic Control
Aalborg, Denmark, June 20-22, 2012

Probabilistic Robust Approach for Discrete Multi-objective Control of Track-Following Servo Systems in Hard Disk Drives

Mohammadreza Chamanbaz * **Ehsan Keikha** **
Venkatakrishnan Venkataramanan *** **Qing-Guo Wang** ****
Abdullah Al Mamun †

* *Data Storage Institute & Electrical and Computer Engineering Department, National University of Singapore (Tell:+6583496560; e-mail: Mohammad_C@dsi.a-star.edu.sg).*
** *Electrical & Computer Engineering Department, National University of Singapore (e-mail: Ehsan.keikha@nus.edu.sg)*
*** *Data Storage Institute (e-mail: Venka_V@dsi.a-star.edu.sg)*
**** *Electrical & Computer Engineering Department, National University of Singapore (e-mail: Elewqg@nus.edu.sg)*
† *Electrical & Computer Engineering Department, National University of Singapore (e-mail: Eleaam@nus.edu.sg)*

Abstract:
This paper deals with the problem of different uncertainties in discrete time track following control of read/write head in hard disk drives (HDD). A multi-objective robust controller is designed which minimizes the worst case root mean square (RMS) value of the positioning error signal (PES) subject to the closed-loop stability in the presence of parametric and dynamic uncertainties. A sequential algorithm based on ellipsoid iteration is utilized to handle parametric uncertainty. Dynamic uncertainty is also represented as linear fractional transformation (LFT) and by the virtue of small gain theorem, the stability of closed-loop system is guaranteed. In this design, two sources of conservatism are avoided: embedding the original non-linear parametric uncertainty into affine structure (converting the original uncertain system into a polytopic uncertain system) and using a single Lyapunov matrix to test all the objectives. The resulting controller has much better track following performance compared to the classical robust approaches which tends to higher storage capacity of HDDs. Simulation as well as experimental results verify the effectiveness of the designed controller.

Keywords: Hard Disk Drive Servo Systems, Track Following, Robust Control, Randomized Algorithms.

1. INTRODUCTION

The amount of data storage worldwide is estimated to be 600 Exabyte which is equivalent to 600 million Hard Disk Drives (HDD) each with a capacity of 1 TB (terabyte). More than 52% of this data storage requirement is met by using HDDs (Hilbert and López [2011]). Continuing trend in the growth of internet, cloud computing and other similar technologies means a growing demand for higher data storage capacity and hence number of HDDs produced. To meet such increasing demand, it is important not only to increase the production volume but also to increase storage density in each HDD. Data density in HDD is defined in units of bits per square inch and commercially produced drives at present have achieved storage density less than 500 gigabits per square inch (Gb/in^2). The HDD industry projects to achieve storage density of 10 Tb/in^2 in the near future.

Data is stored in concentric data tracks on circular disks of magnetic media. Higher storage density implies smaller dimensions for each bit, which requires reduction in distance between adjacent data tracks as well as reduction in length of each bit on a track. Assuming a bit aspect ratio (ratio between bit length and bit width) of 2:1, storage density of 10 Tb/in^2 demands track density of 2,200,000 tracks per inch (TPI). Achievable track density depends on the performance of the head positioning servomechanism of HDD, i.e., how well the read/write head is made to follow the center of a data track. The most important performance measure in HDD servo is Track Misregistration (TMR) which is the variance of the deviation of read/write head from the center of a data track (Mamun et al. [2006]).

Writing and readback of data with bit error rate less than required tolerance level, demands for TMR to be less than 10% of track pitch (distance between the center of two adjacent tracks). This translates into a TMR less than 1.16 nanometer to meet the requirements for 10 Tb/in^2 storage density. Such high performance has to be achieved in a robust manner, that is, for all drives produced in a mass production line. A number of sources of uncertainty in HDD, for example, manufacturing tolerance, change in environmental condition, different raw materials, etc., can contribute to deterioration in performance of HDD servo. We can't expect exactly the same characteristics over a batch of HDDs and changes in system dynamics are unavoidable. These changes can be modeled as uncertainties and a robust controller is required to mitigate those uncertainties. Such approach was not essential in the past when TMR tolerance was not very stringent. The trend in increasing data density now makes use of robust controller essential for HDD.

There have been many attempts to address uncertainty in robust control community. LQG and Kalman filter can be considered as the earliest efforts to address uncertainty; however, the most important breakthrough was the formulation of $\mathcal{H}_\infty$ problem (Zames [1981]). In the subsequent fifteen years, some classical optimal methods were developed such as the idea of structured singular value by Packard and Doyle [1993], Kharitonuv theory of Bhattacharyya et al. [1995] and the optimization technique based on linear matrix inequality LMI(Boyd et al. [1994]). Later on the state and dynamic output feedback design based on multi-objective optimization (Bernstein and Haddad [1989], Scherer et al. [1997]) were introduced; however, in this approach, all Lyapunov matrixes are required to be the same for all the objectives which makes it conservative. The idea of using multiple Lyapunov matrixes in robust design was first proposed in De Oliveira et al. [1999] and the control design based on this approach was presented in De Oliveira et al. [2002]. Most of the approaches which have been mentioned so far, are unable to handle parametric uncertainty directly. A computationally tractable locally optimal output feedback controller is presented in Kanev et al. [2004] which can handle parametric uncertainty. However, computational complexity grows exponentially with the number of uncertain parameters.

To overcome these limitations of robust controller design, a new framework is introduced in robust control which is known as probabilistic approach. There are mainly two controller synthesis techniques in this framework: average performance and robust performance synthesis. The former technique designs the controller such that, with high probability, the absolute error between average performance (with respect to the uncertainty set) and optimal achievable average performance is within a small desired level. This approach is first proposed in Vidyasagar [2001]. The second approach is analogous to worst case design and employs sequential and non-sequential algorithms. Among the sequential algorithms gradient (Calafiore and Polyak [2001]), ellipsoid (Kanev et al. [2003]) and cutting plane (Dabbene et al. [2010]) iterations can be mentioned which can handle convex feasibility problems. Interested readers are referred to Tempo et al. [2005] and Calafiore et al. [2011] for a comprehensive study of probabilistic robust theory.

The main contribution of this paper is to solve the robust multi-objective problem arising from the discrete-time track following dynamic output feedback control of HDD servo system. Sequential algorithm based on ellipsoid iteration is employed in order to efficiently address the problem. Two sources of conservatism, namely embedding the original non-linear parametric uncertainty into affine structure (converting the original uncertain system into a polytopic uncertain system) and using a single Lyapunov matrix to test all the objectives, are avoided in this approach.

Notation

For a square matrix X, $X \succ 0$ ($X \succeq 0$) means X is symmetric and positive definite (semidefinite). I_n denotes $n \times n$ identity matrix. The $\mathcal{H}_\infty$ norm of a transfer function matrix $Q(s) \in \mathcal{RH}_\infty$ is denoted as $\|Q\|_\infty$. The symbol $\lceil x \rceil$ is the minimum integer greater or equal to $x \in \mathbb{R}$. The symbol $\star$ in LMIs denotes entries that come from symmetry. The direct sum of matrixes X_i, $(i = 1, ..., n)$ is denoted as:

$$X_1 \oplus .... \oplus X_n \triangleq diag(X_1, ..., X_n)$$

To indicate that the matrixes (A, B, C, D) are a state space realization of a transfer function matrix $Q(s)$, we use the standard notation

$$Q(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(sI - A)^{-1}B + D$$

2. ROBUST TRACK FOLLOWING CONTROL PROBLEM IN HDD

In this section, formulation of the problem of robust track following control in disk drive servo system is presented. There are parametric as well as non-parametric uncertainties in HDD. The HDD servomechanism uses a dual stage actuator with a voice coil motor (VCM) actuator for coarse movement of the read/write head and piezoelectric micro-actuator for fine positioning of head.

Consider the generalized plant with state space representation

$$P : \begin{pmatrix} z_\Delta \\ z_o \\ y \end{pmatrix} = \left(\begin{array}{c|ccc} A & B_\Delta & B_o & B_u \\ \hline C_\Delta & 0 & D_{\Delta o} & D_{\Delta u} \\ C_o & D_{o\Delta} & D_{oo} & D_{ou} \\ C_y & 0 & D_{yo} & 0 \end{array}\right) \begin{pmatrix} w_\Delta \\ w_o \\ u \end{pmatrix} \quad (1)$$

where u is the control input vector that includes input signals for VCM and micro-actuator, y is the measurement vector which in our problem is positioning error signal (PES), w_o is the vector of exogenous inputs such as airflow vibration and track runout and z_o is the controlled output vector which consists of PES signal. The symbol $\boldsymbol{\Delta}$ represents different uncertainties in the plant model and is described as

$$\boldsymbol{\Delta} \doteq diag\left[\delta_1, \ldots, \delta_n, \boldsymbol{\Delta}_V, \boldsymbol{\Delta}_M\right] \quad (2)$$

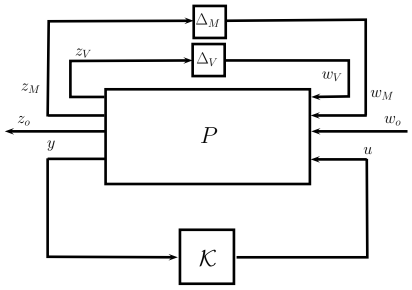

Fig. 1. LFT form of structured uncertainties

where $\delta_i,\ i = 1, ..., n$ are parametric uncertainties such that $|\delta_i| \leq 1$ and $\mathbf{\Delta}_V$ and $\mathbf{\Delta}_M$ are the set of stable rational transfer functions such that $\|\Delta_{V,M}\|_\infty \leq 1$. In the context of HDD, the performance criteria is to minimize root mean square (RMS) of PES signal in time domain while disturbance applied to the system is assumed to be Gaussian white noise. In order to minimize RMS of the PES for the uncertain plant, one should minimize the worst case (over the uncertainty set) $\mathcal{H}_2$ norm of sensitivity transfer function multiplied by disturbance weighting function:

$$minimize\ RMS(PES) \Leftrightarrow \underset{k \in \mathcal{K}}{minimize}\ \underset{\Delta \in \mathbf{\Delta}}{max} \|G_{z_o w_o}(k, \Delta)\|_2 \tag{3}$$

where $G(s) = \mathcal{F}_l(P(s), k(s))$, $\mathcal{F}_l$ represents lower fractional transformation and $\mathcal{K}$ is the set of design parameters. The optimization problem in Eq. 3 is very difficult to solve using classical robust control techniques. In general, there are two approaches for solving this problem in classical robust control. The first approach is mixed $\mathcal{H}_2/\mathcal{H}_\infty$ design (Scherer et al. [1997]) which is unable to solve the problem for the case of parametric uncertainty and hence, plant perturbation can significantly degrade performance of the closed loop system. The second approach is robust $\mathcal{H}_2$ (Kanev et al. [2004]) that can guarantee robust performance for parametric uncertainty but not for dynamic one. In this paper, we employ probabilistic robust technique to cope with parametric uncertainty while dynamic uncertainty is formulated as linear fractional transformation (LFT) and using small gain theorem, stability of the closed loop system against dynamic uncertainty is guaranteed. The uncertainty $\mathbf{\Delta}$ is represented as Fig.1 and the optimization problem of Eq.3 is converted into the following form:

$$minimize\ \ \gamma\ \ subject\ to \begin{cases} \|G_{z_o w_o}(k, \Delta_p)\|_2 < \gamma \\ \|G_{z_V w_V}(k, \Delta_p)\|_\infty < 1 \\ \|G_{z_M w_M}(k, \Delta_p)\|_\infty < 1 \end{cases} \tag{4}$$

where $\Delta_p \in \mathbf{\Delta}_p \doteq diag\,[\delta_1, \ldots, \delta_n]$ is the set of parametric uncertainties. The first constraint in (4) is related to the performance channel which minimizes RMS of the PES signal, second and third constraints guarantee the closed loop stability against dynamic uncertainty. The control algorithm to solve the optimization problem (4) is given in the next subsection.

3. CONTROLLER DESIGN

3.1 Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Controller

In this subsection, formulation and synthesis of mixed $\mathcal{H}_2/\mathcal{H}_\infty$ is discussed which considers simultaneously the robust stability by $\mathcal{H}_\infty$ norm bounds and nominal performance by $\mathcal{H}_2$ norm bound. Parametric uncertainty is considered in the next subsection. In particular, we will study the controller design method based on convex optimization involving LMIs. An LMI based approach is proposed in Scherer et al. [1997] that guarantees quadratic stability of closed loop system. As a single Lyapunov matrix is used to test all objectives, this approach leads to an overly conservative controller. To address this problem De Oliveira et al. [1999] introduced a new approach based on an extra instrumental variables which can be used to build a parameter dependent Lyapunov function. We study LMI based methodologies of mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control design for linear systems using the extended characterization introduced in De Oliveira et al. [2002]. The goal is to design a dynamic output feedback controller in the form of

$$u = k(s)y = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right] y \tag{5}$$

that makes the closed loop system to satisfy (4) but without taking into consideration the parametric uncertainty. This problem is solved by optimization subject to LMI conditions for $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms (see De Oliveira et al. [2002]):

$$\underset{\mathcal{W},\mathcal{P},\mathcal{K},\mathcal{G},\gamma}{minimize}\ \gamma, \quad subject\ to\ \mathcal{M}(\mathcal{W}, \mathcal{P}, \mathcal{K}, \mathcal{G}, \gamma, \Delta_i) \succ 0 \tag{6}$$

where $\mathcal{M}(\mathcal{W}, \mathcal{P}, \mathcal{K}, \mathcal{G}, \gamma, \Delta_i)$ is given by

$$\begin{aligned} &\mathcal{M}(\mathcal{W}, \mathcal{P}, \mathcal{K}, \mathcal{G}, \gamma, \Delta_i) = (\gamma^2 - trace(\mathcal{W})) \oplus \\ &\begin{bmatrix} \mathcal{W} & C^o_{cl}(\mathcal{K}, 0)\mathcal{G} \\ \star & \mathcal{G} + \mathcal{G}^T + \mathcal{P} \end{bmatrix} \oplus \begin{bmatrix} \mathcal{P} & A^o_{cl}(\mathcal{K}, 0)\mathcal{G} & B^o_{cl}(\mathcal{K}, 0) \\ \star & \mathcal{G}^T\mathcal{P}^{-1}\mathcal{G} & 0 \\ \star & \star & I \end{bmatrix} \oplus \\ &\begin{bmatrix} \mathcal{P} & A_{cl}(\mathcal{K}, \Delta_i)\mathcal{G} & B^i_{cl}(\mathcal{K}, \Delta_i) & 0 \\ \star & \mathcal{G}^T\mathcal{P}^{-1}\mathcal{G} & 0 & \mathcal{G}^T C^i_{cl}(\mathcal{K}, \Delta_i) \\ \star & \star & I & 0 \\ \star & \star & \star & I \end{bmatrix} \end{aligned} \tag{7}$$

and $\mathcal{P}^T = \mathcal{P}$ and $\mathcal{W}^T = \mathcal{W}$ are decision variables, $\mathcal{G}$ is a matrix with the same size as $\mathcal{P}$, $i = V, M$ and $A^i_{cl}, B^i_{cl}, C^i_{cl}$ and D^i_{cl} are the closed loop matrixes for the related channels (PES, VCM and secondary actuator see Fig. 1).

3.2 Probabilistic Controller Based on Ellipsoid Iteration

Now we present the probabilistic controller which is the main focus of this paper. There are a number of approaches in control synthesis using probabilistic technique. The earliest systematic method for probabilistic controller synthesis is based on gradient iteration for solving linear quadratic regulator problem presented in Polyak and Tempo [2001]. This approach was later extended to robust

LMIs in Calafiore and Polyak [2001]. A limitation of gradient based method is that it needs a priori knowledge of the radius of the ball which is contained in the feasible set of robust LMI. This radius is used for defining the step size at each iteration. To overcome this problem, iteration method based on ellipsoid algorithm was used in Kanev et al. [2003] which is a localization based method. This approach benefits from convexity of the cost function to be minimized and at each iteration, it shrinks the volume of an ellipsoid to find the probabilistic robust feasible solution.

In our approach, the goal is to minimize $\mathcal{H}_2$ norm of PES subject to stability of the closed loop system in presence of parametric and dynamic uncertainties. The first step in this design is to define a violation function which has the following properties:

(1) $\tau(\theta, \Delta) > 0$ if and only if $\mathcal{M}(\mathcal{W}, \mathcal{P}, \mathcal{K}, \mathcal{G}, \Delta) \not\preceq 0$ for all $\Delta \in \mathbf{\Delta}$ (parametric uncertainty is included).
(2) $\tau(\theta, \Delta) = 0$ otherwise.

A commonly used violation function for such problems is projection of the matrix inequality (7) onto the cone of positive semi-definite matrixes. With this function defined, the original problem (4) is reformulated into the following optimization problem:

$$\theta^* = arg \ \ \underset{\theta \in \Theta}{minimize} \ \underset{\Delta \in \mathbf{\Delta}}{max} \ \tau(\theta, \Delta) \tag{8}$$

where θ^* is any parametrization of the solution set and $\theta \in \Theta$ includes all decision variables.

The design procedure can be divided into two steps: updating design parameters and estimating the level of satisfaction of the violation function for the updated design parameters. The first step is deterministic while the second one is probabilistic. The given flowchart in Fig. 2 illustrates the proposed design procedure. In the first step, the initial ellipsoid $E^0 = \left\{\theta \in \Theta : (\theta - k^0)^T F^{-1} (\theta - k^0) \leq 1\right\}$ with center θ^0 and symmetric positive definite matrix $F \in \mathbb{R}^{N \times N}$ (N is the dimension of the solution set) is chosen such that it includes the solution set. In the next step, the violation function is evaluated for the given design parameters by sampling the uncertainty set based on the given density function f_Δ. The sample bound used is based on log-over-log bound (Tempo et al. [1996]); if the violation function is zero for *all* samples extracted from the uncertainty set, parameters are given as the probabilistic robust feasible solution. Otherwise, the ellipsoid is updated with smaller one which results from the intersection of the current ellipsoid with the half space $H^i = \left\{\theta \in \Theta : \nabla^T \tau(\theta, \Delta)(\theta - \theta^{(i)}) \leq 0\right\}$ where $\nabla^T \tau(\theta, \Delta)$ is the sub-gradient of $\tau(\theta, \Delta)$ defined in Calafiore and Polyak [2001]. The following theorem guarantees the convergence of the algorithm (Kanev et al. [2003]). Before giving the theorem, we state an assumption:

Hypothesis 1. For any design parameter θ which is not in the solution set, there is a nonzero probability to generate a sample $\Delta^{(i)}$ for which $\tau(\theta, \Delta^{(i)}) > 0$.

Theorem 2. If at $(i+1)^{th}$ iteration the ellipsoid is updated as

$$E^{(i+1)} = \left\{\theta \in \Theta : (\theta - \theta^{(i+1)})^T F_{i+1}^{-1} (\theta - \theta^{(i+1)}) \leq 1\right\}$$

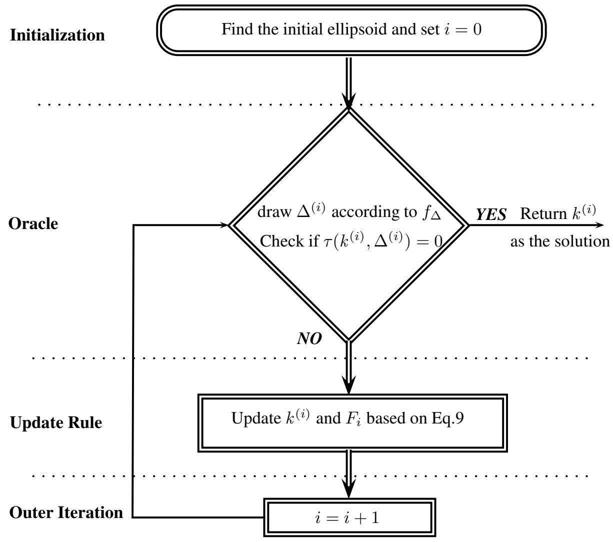

Fig. 2. Probabilistic Design Procedure

where

$$\theta^{(i+1)} = \begin{cases} \theta^{(i)} - \dfrac{1}{N+1} \dfrac{F_i \nabla \tau(\theta^{(i)}, \Delta^{(i)})}{\sqrt{\nabla^T \tau(\theta^{(i)}, \Delta^{(i)}) F_i \nabla \tau(\theta^{(i)}, \Delta^{(i)})}} \\ \qquad if \ \tau(\theta^{(i)}, \Delta^{(i)}) \neq 0 \\ \theta^{(i)} \\ \qquad if \ \tau(\theta^{(i)}, \Delta^{(i)}) = 0 \end{cases}$$

$$F_{i+1} = \begin{cases} \dfrac{N^2}{N^2 - 1} \left(F_i - \dfrac{2}{N+1} \dfrac{F_i \nabla \tau(\theta^{(i)}, \Delta^{(i)}) \nabla^T \tau(\theta^{(i)}, \Delta^{(i)}) F_i^T}{\nabla^T \tau(\theta^{(i)}, \Delta^{(i)}) F_i \nabla \tau(\theta^{(i)}, \Delta^{(i)})} \right) \\ \qquad if \ \tau(\theta^{(i)}, \Delta^{(i)}) \neq 0 \\ F_i \\ \qquad if \ \tau(\theta^{(i)}, \Delta^{(i)}) = 0 \end{cases} \tag{9}$$

and Hypothesis.1 holds, then with probability one the algorithm given in Fig. 2 converges. Furthermore, the maximum number of iterations required to achieve convergence is bounded by $2N \left\lceil \ln \frac{vol(E^0)}{vol(S)} \right\rceil$; where S is the solution set.

Finding the initial ellipsoid that contains the solution set is not a problem. The idea is to find the level set at zero (find θ such that $\tau(\theta, 0) = 0$) and bound the level set with a hyper-rectangle, and finally, find an ellipsoid which encircles the hyper-rectangle.

4. SIMULATION STUDY

This section presents simulation results of the controller which is designed for a dual stage servo of an HDD available in the market.

4.1 Generalized Plant

We use frequency domain system identification, a widely used technique, for modeling the dual stage actuator. In this experiment, the input to VCM or micro-actuator is subject to a sweep sine signal generated by Dynamic Signal Analyzer [1] (DSA), and the displacement of the read/write head is measured using a Laser Doppler Vibrometer [2] (LDV). For the VCM, the gain decreases at

[1] HP 35670A, Hewlett Packard Company, Washington.
[2] Polytec OFV 5000, Polytec, Waldbronn, Germany.

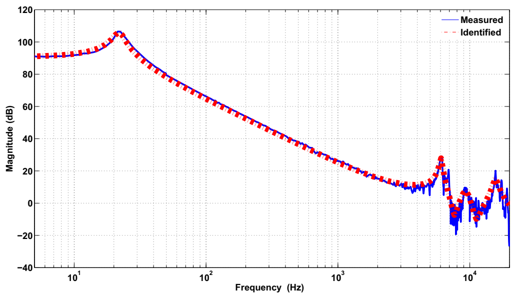

Fig. 3. Measured as well as identified frequency response for VCM actuator

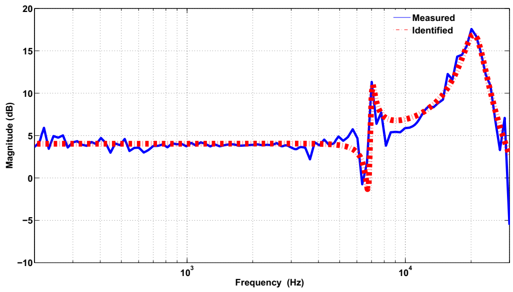

Fig. 4. Measured as well as identified frequency response for micro-actuator

the rate of 40 dB/decade with increasing frequency. For identification of VCM model, different amplitudes are used in different ranges of frequencies to get good response in the wide range of frequency of interest. The measurement condition is as close as possible to the normal operating condition of disk drive with the disk rotating at the nominal speed of 7200 rpm. However, the displacement is measured using external sensor (LDV) as the signals from the proprietary PES demodulator chip of HDD are not accessible. In order to make the laser beam of LDV shine the read/write head slider, a small lateral opening is made on the casing of HDD. The opening is then covered with a piece of glass to keep the drive enclosure free from dust and other pollutants. Covering the opening also ensures that the circulation of air inside the drive enclosure is minimally affected. The frequency response obtained from these experiments and the response of the model identified are shown in Fig. 3 and Fig. 4 for VCM and micro-actuator, respectively. The transfer function of VCM and micro-actuator are in the form of:

$$G_{VCM} = \sum_{i=1}^{4} \frac{A_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}$$

$$G_{MA} = \sum_{i=1}^{2} \frac{A_{mi}}{s^2 + 2\zeta_{mi} \omega_{mi} s + \omega_{mi}^2}$$

where parameters $\omega_i, \zeta_i, A_i, \omega_{mi}, \zeta_{mi}$ and A_{mi} are plant parameters.

The next step in controller design is to augment the plant with necessary weighting functions. Since the problem is of regulation type, the output sensitivity curve is of

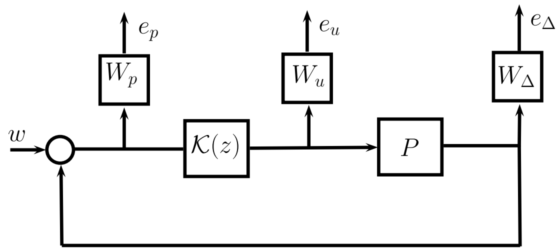

Fig. 5. Augmented Plant

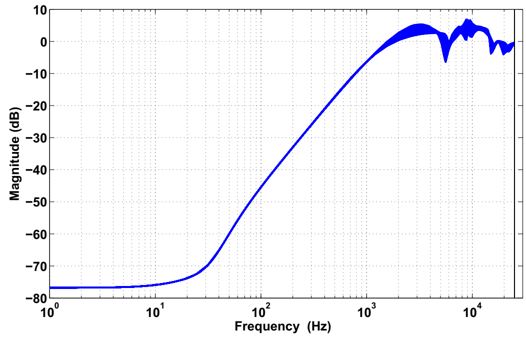

Fig. 6. Sensitivity function for the designed controller

	ζ	ω
G_{VCM}	10%	5%
G_{MA}	10%	6%

Table 1. Parameter Variation In The Plant

Gain Margin (dB)		Phase Margin (°)	
Nominal	Worst Case	Nominal	Worst Case
8.6	*5*	*45.5*	*31.85*

Table 2. Comparison of the Nominal and Worst Case Stability Margins

vital importance. Then, the open loop plant is augmented as Fig. 5. The weighting function W_p is introduced to include the frequency dependent weighting of the desired sensitivity function and, generally speaking, the sensitivity function should follow $\frac{1}{W_p}$ as closely as possible. The function W_u is used to penalize amplitude of the control signal applied to the plant; it represents a vector with separate penalty function for VCM and micro-actuator. Finally, the function W_Δ defines the dynamic uncertainty considered for the plant; it is small (less than 20%) at low frequencies and increases with increasing frequency (larger than 200% at high frequencies).

4.2 Performance Verification

The control algorithm given in Section 3 is simulated using MATLAB (Tremba et al. [2008]). The number of uncertain parameters considered is 10; ranges of their variations are given in Table. 1. We assume uniform distribution while sampling the uncertainty set due to its worst case nature. To test robustness of the designed controller, we use 100 randomly chosen uncertain plants and apply the controller to each of these plants. The sensitivity function for each

TMR (*nm*)		RMS(u_V) (mV)		RMS(u_M) (mV)		$\|u_V\|_\infty$(mV)		$\|u_M\|_\infty$(mV)	
Nominal	Worst Case	Nominal	Worst Case	Nominal	Worst Case	Nominal	Worst Case	Nominal	Worst Case
8.11	*8.29*	*8.2*	*8.4*	*244*	*254*	*35.4*	*35.4*	*1118*	*1148*

Table 3. Comparison of the Nominal and Worst Case Performance Specifications

closed-loop system is given in Fig. 6. It is clear from the figure that the sensitivity functions do not disperse significantly in presence of real parametric variations. Furthermore, stability margins (gain and phase margins) were computed for each of the randomly picked plant transfer functions. Table. 2 shows the nominal as well as worst case stability margins for the designed controller. Similar analysis with an $\mathcal{H}_2$ controller designed for the nominal plant shows that for some instances of uncertainty set, the closed loop becomes unstable. It clearly proves the necessity of robust controller design for the disk drive servo design. Also a modified version of the standard disturbance model which has been used in the literature is used in order to evaluate the performance of the designed controller in rejecting disturbances. The achieved track misregistration (TMR), RMS and peak values of the control input signals for the nominal as well as worst case scenarios are given in Table. 3. As it can be seen, they do not change significantly for different scenarios of the uncertain plant. Therefore, the designed controller satisfies the performance requirements in a robust manner.

5. CONCLUSION

A discrete time multi-objective robust controller is designed using probabilistic robust approach to handle different parametric and dynamic uncertainties in disk drive. An iterative algorithm based on ellipsoid iteration is used to find the probabilistic robust feasible solution. The design is less conservative compared to classical robust approaches in a way that the original non-linear parametric uncertainty is treated as it is and is not embedded into affine structure. Furthermore, multiple Lyapunov matrixes are used for different objectives. In disk drive, performance is of great importance; the control algorithm, while handling different uncertainties, should not be conservative. In this design, by allowing a small probability that the objective function being violated, the performance of the resulting controller is close to the nominal controller. Effectiveness of the design algorithm is also verified through experiment.

REFERENCES

D. S Bernstein and W. M Haddad. LQG control with an H_∞ performance bound: a riccati equation approach. *IEEE Transactions on Automatic Control*, 34(3):293–305, March 1989.

S. P. Bhattacharyya, H. Chapellat, and L. H. Keel. *Robust Control: The Parametric Approach*. Prentice Hall, Upper Saddle River, 1995.

Stephen Boyd, Laurent El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM, Philadelphia, 1994.

G. Calafiore and B. T Polyak. Stochastic algorithms for exact and approximate feasibility of robust LMIs. *Automatic Control, IEEE Transactions on*, 46(11): 1755–1759, 2001.

Giuseppe C. Calafiore, Fabrizio Dabbene, and Roberto Tempo. Research on probabilistic methods for control system design. *Automatica*, In Press, Corrected Proof, 2011.

F. Dabbene, P. S. Shcherbakov, and B. T. Polyak. A randomized cutting plane method with probabilistic geometric convergence. *SIAM Journal on Optimization*, 20(6):3185, 2010.

M. C. De Oliveira, J. Bernussou, and J. C. Geromel. A new discrete-time robust stability condition. *Systems & Control Letters*, 37(4):261–265, July 1999.

M. C. De Oliveira, J. C. Geromel, and J. Bernussou. Extended H_2 and H_∞ norm characterizations and controller parametrizations for discrete-time systems. *International Journal of Control*, 75(9):666, 2002.

Martin Hilbert and Priscila López. The world's technological capacity to store, communicate, and compute information. *Science*, 332(6025):60 –65, April 2011.

S. Kanev, B. De Schutter, and M. Verhaegen. An ellipsoid algorithm for probabilistic robust controller design. *Systems & Control Letters*, 49(5):365–375, August 2003.

S. Kanev, C. Scherer, M. Verhaegen, and B. De Schutter. Robust output-feedback controller design via local BMI optimization. *Automatica*, 40(7):1115–1127, July 2004.

Abdullah Al Mamun, GuoXiao Guo, and Chao Bi. *Hard Disk Drive: Mechatronics and Control*. CRC Press, 1 edition, November 2006.

A. Packard and J. Doyle. The complex structured singular value. *Automatica*, 29(1):71–109, January 1993.

B. T. Polyak and R. Tempo. Probabilistic robust design with linear quadratic regulators. *Systems & Control Letters*, 43(5):343–353, August 2001.

C. Scherer, P. Gahinet, and M. Chilali. Multiobjective output-feedback control via LMI optimization. *Automatic Control, IEEE Transactions on*, 42(7):896–911, 1997.

R. Tempo, E. W Bai, and F. Dabbene. Probabilistic robustness analysis: explicit bounds for the minimum number of samples. In *, Proceedings of the 35th IEEE Decision and Control, 1996*, volume 3, pages 3424–3428 vol.3. IEEE, December 1996.

R. Tempo, G. Calafiore, and F. Dabbene. *Randomized Algorithms for Analysis and Control of Uncertain Systems*. Springer-Verlag London Limited, 2005.

A. Tremba, G. Calafiore, F. Dabbene, E. Gryazina, B. Polyak, P. Shcherbakov, and R. Tempo. RACT: randomized algorithms control toolbox for MATLAB. In *Proc. 17th World Congress of IFAC, Seoul*, page 390–395, 2008.

M. Vidyasagar. Randomized algorithms for robust controller synthesis using statistical learning theory. *Automatica*, 37(10):1515–1528, October 2001.

G. Zames. Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximate inverses. *Automatic Control, IEEE Transactions on*, 26(2):301–320, 1981.